

Department of Mathematics and Actuarial Science. Special Functions, Partial Differential Equations and Harmonic Analysis, a conference in honor of Calixto P. Calderón

November 16, 17 & 18, 2012.

Main Speakers.

n-th Riemann derivatives, *n*-th quantum derivatives, and further generalizations.

Marshall Ash, DePaul University

Abstract: Here are three different procedures for finding the second derivative of a smooth function at a real nonzero number x,: evaluate the limit as $h \to 0$ of $\frac{f(x+h)-2f(x)+f(x-h)}{h}$, evaluate the limit as $q \to 1$ of $\frac{f(qx)-(1+q)f(x)+qf(x/q)}{((q-1)x)^2}$, and evaluate the limit as $h \to 0$ of $\frac{2(f(x+h)-f(x)-f'(x)h)}{h^2}$. These procedures respectively motivate the definitions of the symmetric nth Riemann derivative, the symmetric nth Quantum Riemann derivative, and the nth Peano derivative. The first two of this trio admit even broader generalization. All of these generalizations appear to be generically equivalent. We will survey the known results in this area.

Positive sums of Jacobi polynomials.

Richard Askey, University of Wisconsin

Abstract: Starting with Fejer's work on (C,1) summability of Fourier series and earlier work of Stieltjes on Gaussian quadrature, there has been interest in positive polynomials, and in particular when certain sums of Jacobi polynomials are positive. I used to think that some deep results of George Gasper gave an adequate description of what we could expect. However, a new positive sum arose in some applied work of Joachim Schoberl as a conjecture which was later proven by Veronica Pillwein. This example shows that the inequalities Gasper and I studied need to be combined with this new result, at least in the case of symmetric Jacobi polynomials. A summary of these inequalities will be given.

A happy mathematical relation.

Alexandra Bellow, Northwestern University

Abstract: This talk is about the mathematical relationship of the brothers Alberto and Calixto Calderón

Remarks on some of the beautiful work of Professor Calixto Calderón and a selection of our shared interests within harmonic analysis.

Robert Fefferman, University of Chicago

Abstract: It is perhaps not so surprising that Professor C.P. Calderón's interests within harmonic analysis and mine overlap strongly. What has been quite striking to me, however, is the existence of a selection of his work and mine that relate so very directly. I shall discuss this overlap, and it will involve a tour of some topics in the theory of singular integrals and maximal functions in several variables.

An Extension of Widders Inequality.

Constantine Georgakis, DePaul University

Abstract: Widder's inequality is a refinement of Hilbert's inequality. That is, if $\{a_n : n \geq 0\}$ is a sequence in l^2 , then

$$\sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{a_m \overline{a_n}}{(m+n+1)} < \pi \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} {m+n \choose m} \frac{a_m \overline{a_n}}{2^{m+n+1}} < \pi ||a||_2,$$

where the constants are best possible and the strict inequality holds unless the sequence $\{a_n : n \geq 0\}$ is null.

The continuity properties of the sequence-to-function Hausdorff transformation are used and an extension of Widder's inequality is presented, which includes as a special case the following inequality for the Hilbert matrix transform, the transformation of Abel means and the Borel transform of a sequence. If $\{a_n : n \geq 0\}$ in l^p and 1 , then

$$\left[\sum_{n=0}^{\infty} \left| \sum_{m=0}^{\infty} \frac{a_m}{(m+n+1)} \right|^p \right]^{1/p} < \Gamma(\frac{1}{p}) \left[\sum_{n=0}^{\infty} \left| \sum_{m=0}^{\infty} a_m x^m \right|^p (1-x)^{p-2} \right]^{1/p}
< \pi \csc \frac{\pi}{p} \left[\int_0^{\infty} \left| \sum_{m=0}^{\infty} a_m e^{-y} \frac{y^m}{m!} \right|^p \right]^{1/p} < \pi \csc \frac{\pi}{p} ||a||_p,$$

where the constants are best possible and the strict inequality holds unless the sequence $\{a_n : n \geq 0\}$ is null.

A case study for critical non-linear dispersive equations: the energy critical wave equation.

Carlos Kenig, University of Chicago

Abstract: In this lecture we will illustrate some recent developments in the theory of nonlinear dispersive equations and forecast some possible future developments, by means of one example, namely the energy critical wave equation in three space dimensions. The issues studied are global existence, finite time blow-up, scattering and soliton resolution.

Symbolic Calculus for Singular Integral Operators on C^1 manifolds and C^1 Curvilinear Polygons.

Jeff Lewis, University of Illinois at Chicago

Abstract: A Multilinear Commutator Theorem of Alberto Calderón, Calixto Calderón, Eugene Fabes, Max Jodeit, and Nestor Rivière can be used to justify the symbolic calculus for singular integral operators (S.I.O.) with C^0 coefficients on C^1 manifolds and C^1 Curvilinear Polygons. In particular a system of S.I.O. is Fredholm on L^p if and only the symbol is a nonsingular homomorphism. For C^1 curvilinear polygons, the symbol is constructed using the Mellin transform and can be extended to handle power weights near the vertices. The talk is a review of work with Renata Selvaggi and Irene Sisto and work with Cesare Parenti.

Sobolev Inequalities: Towards a unified theory using rearrangements, interpolation and isoperimetry.

Mario Milman, Florida Atlantic University

Abstract: I am very happy and proud to be part of this celebration. I owe a great deal to Calixto and he has been a very good friend over many years. Indeed, the motivation for this work goes back more than 30 years ago to a joint paper with Calixto on real interpolation of Sobolev spaces. I would briefly like to place my early work with Calixto in the context of this talk. The classical interpolation theorems (Riesz, Marcinkiewicz, Zygmund, Cotlar,....) all had a similar objective: from a pair of initial estimates one derives intermediate results for spaces of a particular type. In particular, the type of estimates, or equivalently the spaces that intervene in the inequalities, that one is trying to prove are known or guessed in advance. This point of view was fundamentally changed by Alberto Calderón, who together with his student Evelio Oklander (U. Chicago, 1964), and independently from everyone else, invented the real method of interpolation. In Alberto Calderóns method, from a pair of initial estimates, one derives a rearrangement inequality that characterizes all the results obtainable by interpolation. This is the point of view that Calixto and I adopted as well. Now, while interpolation

results are very useful it turns that, in many in-stances, Sobolev inequalities (of first order) can be extrapolated. For example: it is well known that ONE L^p Sobolev embedding frequently implies a family of such inequalities and, moreover, that some extreme inequalities (e.g. the Gagliardo-Nirenberg inequality) imply all such L^p Sobolev inequalities. This follows from the scale properties of the L^p spaces (Hölder), and the scaling of the inequalities that results of using the chain rule when one replaces f by —f—? . But what to do for more general spaces? Obstacles: What can one do about the powers, the chain rule, the scaling of the L^p -s. Moreover, Sobolev embeddings can change dramatically when we consider different geometries. For example, while the gain of integrability in Euclidean Sobolev inequalities depends on the dimension, that is not the case in Gaussian inequalities. In particular, as is well known, the spaces one uses for one theory are different from the other. In this talk I will discuss new methods to derive all Sobolev inequalities starting from a known given one, or equivalently assuming that one knows the isoperimetric profile associated with a given geometry. In the spirit of Alberto Calderón ideas we characterize the inequalities using rearrangement inequalities that now involve the gradient or a modulus of continuity as well as the isoperimetric profile associated with the geometry. As the isoperimetric profile changes the inequalities are automatically updated and provide the framework for sharp embeddings. This leads to a reformulation, as well as an extension, of the classical equivalence between the Gagliardo-Nirenberg inequality and the isoperimetric inequality. In this fashion the isoperimetric profile associated with a given geometry determines the corresponding Sobolev inequalities. For example, in the Euclidean case we recover the usual Sobolev inequalities while for Gaussian measure we obtain logarithmic Sobolev inequalities. Using suitable variants of Peetres K- functional we also obtain new fractional Besov-Sobolev inequalities and formulate a generalized Morrey-Sobolev theorem. This represents joint work with Joaquim Martin.

Riemann Redux.

Yoram Sagher, Florida Atlantic University

Abstract: The change of variable formula for the Henstock-Kurzweil integral will be proved. Joint work with V. Rutherfoord.

Local sharp maximal functions.

Alberto Torchinsky, Indiana University

Abstract: In considering the problem of the resistance of materials to certain types of deformations, F. John was led to the study of quasi-isometric mappings. Thus the space of functions of bounded mean oscillation (BMO) was

introduced and the John-Nirenberg inequality established. J.-O. Strömberg adopted this setting when studying spaces close to BMO and considered different ways of describing the oscillation of a function f on cubes. The local maximal functions of Strömberg are of particular interest because they allow for pointwise estimates for Calderón-Zygmund singular integral operators.

We consider properties of medians as they pertain to the continuity and vanishing oscillation of a function. Our approach is based on the observation that (maximal) medians are related to local sharp maximal functions restricted to a cube $Q_0 \subset \mathbb{R}^n$ with parameter $0 < s \le 1/2$ by means of the expression

$$M_{0,s,Q_0}^{\sharp,\phi}f(x) = \sup_{x \in Q, Q \subset Q_0} \inf_{c} \frac{m_{|f-c|}(1-s,Q)}{\phi(|Q|)}$$
$$\sim \sup_{x \in Q, Q \subset Q_0} \frac{m_{|f-m_f(1-s,Q)|}(1-s,Q)}{\phi(|Q|)},$$

where $\phi=1$ in the case of functions with bounded median oscillation with parameter s (bmo_s) , and satisfies appropriate conditions in the case of functions with vanishing median oscillation with parameter s (vmo_s) . Strömberg showed that $bmo_s=BMO$, and, similarly, for sufficiently small $s, vmo_s=VMO$, the space of functions of vanishing mean oscillation. Moreover, since VMO is known to contain bounded discontinuous functions, we complete the picture by giving criteria for continuity for functions equivalent to a bounded function on a cube in terms of medians.

Finally, in the spirit of the "local mean oscillation" decomposition of Lerner we generate two "local median oscillation" decompositions.

Recent results about commutators of multilinear singular integrals.

Rodolfo Torres, University of Kansas

Abstract: We will present some results from two collaborations, one with C. Peréz, G. Pradolini and R. Trujillo-González and another one with A. Bényi, which describe properties of commutators of multilinear singular integrals with point-wise multiplications by functions in the space of functions of bounded mean oscillation, or an appropriate subspace of it. The results put in evidence one more time the subtle cancellation present in commutators of this type, which lead to "better than expected" estimates.

Weighted norm estimates for rough Calderón-Zygmund homogeneous singular integrals; Regularity results for solutions of some smooth infinitely degenerate quasilinear equations.

Richard Wheeden, Rutgers University

Abstract: The talk will be divided into two parts. The first part reviews some joint work with David Watson about one-weight norm estimates for homogeneous CZ singular integrals whose kernels are merely of class $L\log L$ on the unit sphere. The method is largely inspired by results of Calixto Calderón from 1973 about the starlike maximal function. The second part, which is joint work with Cristian Rios and Eric Sawyer, is about C-infinity regularity of solutions of a class of infinitely degenerate second order quasilinear equations which have C-infinity data. The two parts are unrelated, and major open questions remain about both parts.

Chromatic Derivatives and Their Applications.

Ahmed Zayed, DePaul University

Abstract: A new notion of derivative, called chromatic derivative, has recently been introduced with applications in communication engineering. These derivatives lead to series expansions of analytic functions like Taylor series but with better convergence properties. In this talk we shall give an overview of chromatic derivatives and series and some of their applications.

Inequalities in L^{p-1} for the extended L^p best approximation operator.

Felipe Zo, Universidad Nacional de San Luis

Abstract: The best polynomial approximation operator was recently extended from L^p to $L^{p?1}$. We study weak and strong inequalities for maximal operators related with the extended best polynomial approximation operator. As an application we obtain norm convergence of the coefficients of the best polynomial approximation.

Calixto's Ph.D. students.

Current Emerging Paradigms, Frameworks, Approaches, and Insights in the Modeling of Cancer.

Evans Afenya, Elmhurst College

Abstract: As biomedicine becomes increasingly quantitative in scope and content and various challenges are encountered in the prevention, detection,

treatment, and management of the cancers, mathematical models correspondingly assume importance in synthesizing and comprehending some of the dynamics underlying the behavior of cell aggregates and systems. Within this framework, diverse approaches are adopted for obtaining some models that describe the development and propagation of malignancy in the disease state. Current emerging paradigms that seek to define new ways and directions for looking at all the cancers in the face of the continuing challenges brought on by such diseases are discussed and placed within mathematical modeling settings. Various techniques are employed in analyzing the models and biomedical insights that they engender are discussed and placed in relevant context. Predictions offered by the models are then considered and conclusions are drawn.

Modeling aneurysm evolution, growth and rupture.

Tor A Kwembe, Jackson State University

Abstract: This talk will focus on a mathematical model describing the evolution, growth and risk rupture of aneurysms contained within a cylindrical region of blood vessels-fusiform aneurysms. The model describes a quasi-static, non-convectional acceleration, axi-symmetric Navier-Stokes equations in cylindrical coordinates coupled with the Camenschi-Fung type linear elastodynamic system of equations with filtration. Analytical and numerical solutions will be discussed. Results affirmed that the intra-aneurysmal pressure and bloodstream flow account for the evolution and growth of aneurysms, and we find that an aneurysm may rupture when the ratio of the lateral membrane contraction to longitudinal membrane extension approaches one. Numerical properties of intra-aneurysmal pressure, impact fluid velocity, membrane displacement and the deformed radius with respect to the Poisson ratio, membrane thickness and extensional rigidity are discussed. The importance of the findings is rested on the fact that they can be used to improve noninvasive means for predicting aneurysm rupture, and treatment and management decisions after rupture.

Spanish Calculators and Galileo.

Myrna La Rosa, Triton College

Abstract: In this talk, I will show the impact of the Parisian School of Physico-Mathematics in Collegio Romano which consequently reached Galileo. The Physico-Mathematics initiated by the Merton College Masters, Oxford circa 1330 was expanded through the colleges of the University of Paris. The title of this talk, Spanish Calculators and Galileo, tells the main actors to whom this linkage between University of Paris and Collegio Romano is due. I claim that Domingo de Soto, Spanish Calculator, was the first one, as far as we know, who stated the law of falling bodies.